Winter 2024 **Math 597: Real Analysis** MWF 11:00AM – 12:00 PM,

Instructor: Zaher Hani, (*Office:* 5834 East Hall (EH)), *Email:* zhani@umich.edu, *Website:* https://sites.lsa.umich.edu/zhani/.*Office hours:* MW 1:30pm-2:30pm.

Teaching Assistent: Jasper Liang. Email: jspliang@umich.edu. Office Hours: TBA.

Course Coordinates: MWF 11:00AM–12:00 pm in 1084 East Hall.

Prerequisites: The course assumes a solid knowledge of proof-based undergraduate real analysis in single and several variables (Advanced Calculus; MATH 451 or 295 or 297) and linear algebra (MATH 217 or 296). A mathematical maturity comparable to that acquired from taking several advanced abstract math courses is assumed. The student should be familiar with mathematical logic, some basic set theory, and basic point-set topology (at least metric spaces). MATH 590 (Introduction to Topology) is encouraged before taking this course. See Chapter 0 of Folland for a quick review of some of the latter concepts.

Textbook:

- 1. (**Required**) Folland. *Real Analysis: Modern Techniques and Their Applications*. 2nd Edition. ISBN 978-0471317166.
- 2. (Optional) Stein and Shakarchi, *Real Analysis: Measure Theory, Integration, and Hilbert Spaces.* Princeton Lectures in Analysis, First Edition. ISBN 978-0691113869.
- 3. (Optional) Tao, An Introduction of Measure Theory. An online version can be found here https://terrytao.wordpress.com/books/an-introduction-to-measure-theory/

Homework: There will be (roughly) weekly homework sets, that are due on Thursday. All homework should be scanned and submitted on Gradescope. Late submissions are not allowed, except under compelling reasons. You are encouraged to discuss the problems together, but each has to submit their own written version of the solution.

Grading: Grading will be based on homework (50%), a midterm exam (20%), and a final exam (30%). All exams are in-person^{*}, and closed book. No notes or electronics are allowed during

^{*}The University of Michigan is committed to providing equal opportunity for participation in all classes, programs, services and activities. Requests for accommodations by persons with disabilities may be made by contacting the Services for Students with Disabilities (SSD) Office located at G664 Haven Hall. The SSD phone number is 734-763-3000 and their website is ssd.umich.edu. Once your eligibility for an accommodation has been determined, please share this information with me at the beginning of the term, or at least two weeks before you need the accommodation for an exam, project, etc.

the exam. The dates and time of the exams are written below under Important Dates. Make sure you clear your schedules for them.

Course Description: This is one of the basic courses for students beginning study towards the Ph.D. degree in mathematics. Highly advanced undergraduate math students and students from other fields may take this course but they should expect that the workload is heavy and the pace is fast.

The approach is theoretical and rigorous and emphasizes abstract concepts and proofs. The topics covered include abstract measure spaces, Lebesgue measure on Euclidean space, Lebesgue integration theory, Convergence theorems, Introduction to L^p spaces, Lebesgue differentiation theorem.

Outline: Here is a rough outline of the content of this course. The ordering of topics might change.

- A) Measures
 - The problem of measure
 - Definition of abstract measures: σ -algebras, and measure spaces
 - Outer measures
 - Construction of Lebesgue measure on Euclidean space.
- B) Lebesgue Integration theory
 - Measurable functions
 - Integration of nonnegative functions
 - Integration of complex functions
 - Convergence theorems
 - Product measures

C) Introduction to L^p spaces

- Banach and Hilbert spaces
- L^p spaces and completeness
- The L^2 Hilbert space
- D) Signed measures and differentiation
 - Signed and complex measures
 - The Lebesgue-Radon-Nikodym theorem
 - The Hardy-Littlewood Maximal function and Lebesgue's differentiation theorem

Important Dates

Jan 10	First day of classes
Jan 15	MLK Day, no class
Jan 30	Drop/Add Deadline
Feb 26- March 3	Winter Break- No Class
March 6 (tentative)	Midterm Exam (6:00-8:00pm)
April 22	Last day of class.
April 25	Final Exam $(4:00-6:00 \text{pm})$