Winter 2022

Math 657: **Nonlinear Partial Differential Equations**

MW 2:30 – 4:00 PM,

Instructor: Zaher Hani, (*Office:* 5834 East Hall (EH)), *Email:* zhani@umich.edu,
Website: https://sites.lsa.umich.edu/zhani/. *Office hours:* TBA

Prerequisites: Ideal requirements for this course would be Math 656 and a solid background in graduate real analysis. Please consult instructor otherwise.

Course Coordinates: MW 2:30–4:00 pm in 4096 East Hall.

Textbook: No textbook is required.

Reference and resources: Here are some references for the course. The first four pertain to the PDE content, and the last three pertain to the prerequisite Fourier Analysis background.

1. Terry Tao's lecture notes on Fluid Equations available of his "What's new" blog: https://terrytao.wordpress.com/category/teaching/254a-incompressible-fluid-equations.
2. L. C. Evans, *Partial Differential Equations.* Graduate Studies in Mathematics.
3. A. Majda, A. Bertozzi, *Vorticity and incompressible flow.* Cambridge texts in Applied Mathematics. ISBN: ISBN 0 521 63948 4.
4. T. Tao, *Nonlinear Dispersive Equations: local and global analysis.* CBMS Regional Conference Series in Mathematics. ISBN: 978-0-8218-4143-3.
5. H. Bahouri, J-Y. Chemin, R. Danchin, *Fourier Analysis and Nonlinear Partial Differential Equations.* Springer. ISBN 978-3-642-16829-1.
6. J. Duoandikoetxea , *Fourier Analysis.* Graduate Studies in Mathematics, AMS. ISBN: 978-0821821725.
7. E. Stein and G. Weiss, *Introduction to Fourier Analysis on Euclidean Spaces.* Princeton University Press. ISBN-13: 978-0691080789

Homework: There will be a few homework sets throughout the semester.

Course Description: Partial differential equations are at the core of models in science, engineering, economics, and related fields. These equations and their solutions have interesting structures that are studied by a beautiful combinations of methods from analysis, geometry, probability and other mathematical fields.

This course is an introduction to nonlinear partial differential equations for a diverse audience in pure and applied mathematics, but with a focus on rigor and developing analytical proofs and techniques. It will start by reviewing and introducing some analytical techniques that are central to the modern study of nonlinear PDEs like Fourier analysis, Sobolev spaces, and Littlewood-Paley theory. Afterwards, the course will present various techniques to construct and analyze solutions of nonlinear PDE. The focus will be on models coming from fluids (Euler and Navier-Stokes equations) and nonlinear dispersive/hyperbolic equations (the nonlinear Schrödinger and wave equations). Other topics and equations will be covered time permitting.

Outline: The following is a rough outline of the content of this course. The amount of content covered in Item C) below might vary depending on how deep we go in our study of items A) and B).

A) Harmonic Analysis Prerequisites

- Introduction to the Fourier transform (prerequisite or study Canvas notes on your own)
- Introduction to Littlewood-Paley theory
- Sobolev spaces and embeddings

B) Introduction to Fluid Equations

- The Navier-Stokes and Euler's equations: basic properties
- Construction of solutions via energy methods
- Weak solutions
- Euler's equation: Existence theory and blowup criteria

C) Introduction to nonlinear dispersive/hyperbolic equations (time permitting)

- What are dispersive PDE: Basic features and properties
- Linear Equations and their estimates
- Nonlinear theory: local vs global existence.

Important Dates

Jan 4	First day of classes
Jan 16	MLK Day, no class
Feb 27- March 5	Winter Break- No Class
April 17	Last day of class.